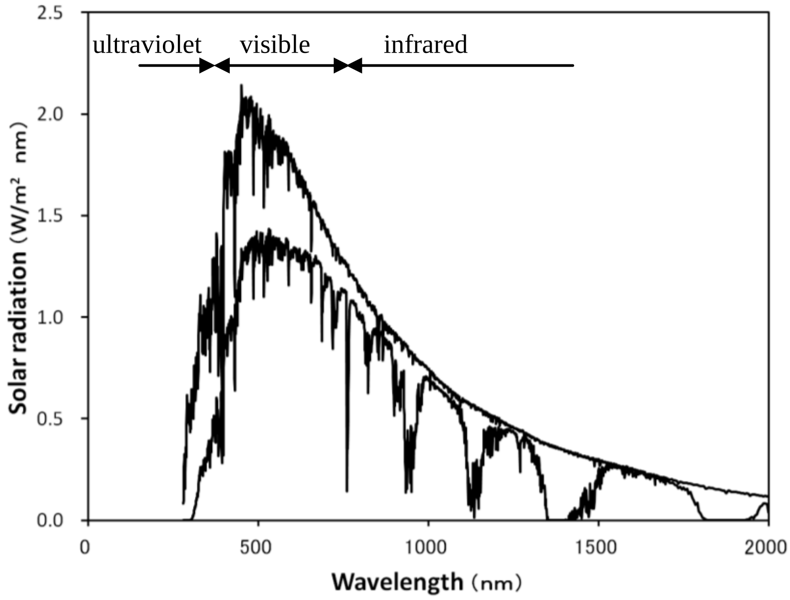
The heat budget of the atmosphere and the greenhouse effect

1. Solar radiation

1.1 Solar constant

The radiation coming from the sun is called **solar radiation** (**shortwave radiation**). Most of the solar radiation is **visible light** (with wavelength of 380 to 770 nm) (Note that 1000 nm = 1 μm = 0.001 mm).



(The National Renewable Energy Laboratory)

Figure 1: Spectrum of solar radiation

(The top line indicates values at the top of the atmosphere, while the bottom one those at the ground surface.)

The total amount of solar radiation per unit area is about 1370 W/m^2 at the top of the atmosphere. This value is defined as **solar constant** (hereinafter referred to as S_0). At the mean distance of the earth from the sun ($\cong 1.5\times 10^{11}$m), the solar constant is $S_0 = 1.37\times 10^3 \mathrm{W/m^2}$.

1.2 Shadow area of a planet

The shadow area of the earth's disk which intercepts the solar radiation is given by πR^2 where R is the earth's radius, while the surface area of the earth is $4\pi R^2$.

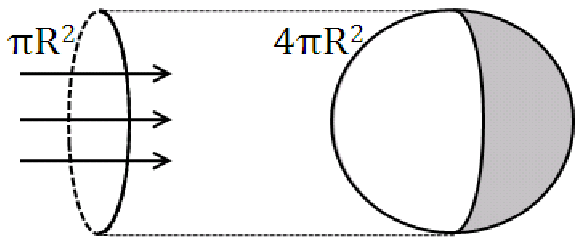


Figure 2: Shadow area and surface area

Therefore, averaging the total incoming solar radiation over the earth's surface, it is reduced to a quarter of the solar constant ($\cong 340\ \mathrm{W/m^2}$).

$$S_0 \quad \rightarrow \quad \frac{S_0 \times \pi R^2}{4\pi R^2} = \frac{S_0}{4}$$

1.3 Planetary reflectivity

In addition, we must consider the fact that not all of the solar radiation incident on the earth is absorbed. In fact, about 30 % of the solar radiation is reflected back to space. This reflectivity is called **albedo** (α). The earth's albedo is 0.30. Thus, the earth absorbs only 70 % of the solar radiation. Therefore, the net incoming solar radiation averaged over the earth's surface is

$$S = (1-\alpha) \times \frac{S_0}{4} = \frac{(1-\alpha)S_0}{4} \tag{1}$$

For the present earth, $S \cong 2.4 \times 10^2\ \mathrm{W/m^2}$.

2. Terrestrial radiation

The earth always absorbs the solar radiation. Nevertheless, the mean temperature of the surface or the atmosphere is almost constant. It is because the earth radiates as much energy as it receives. The radiation emitted from the earth to space is referred to as **terrestrial radiation** (**longwave radiation**). Most of the terrestrial radiation is emitted as **infrared light**. In general, an object emits electromagnetic wave such like infrared radiation, corresponding to its temperature. This radiation is called **blackbody radiation**. The power of blackbody radiation F can be written

$$F = \sigma T^4 \tag{2}$$

where T is temperature (absolute temperature; 0℃=273.15K) of an object. σ is the **Stefan-Boltzmann constant**, and has a value of

$\sigma = 5.67 \times 10^{-8} \mathrm{W/m^2K^4}$. This relationship is known as **Stefan-Boltzmann's law.**

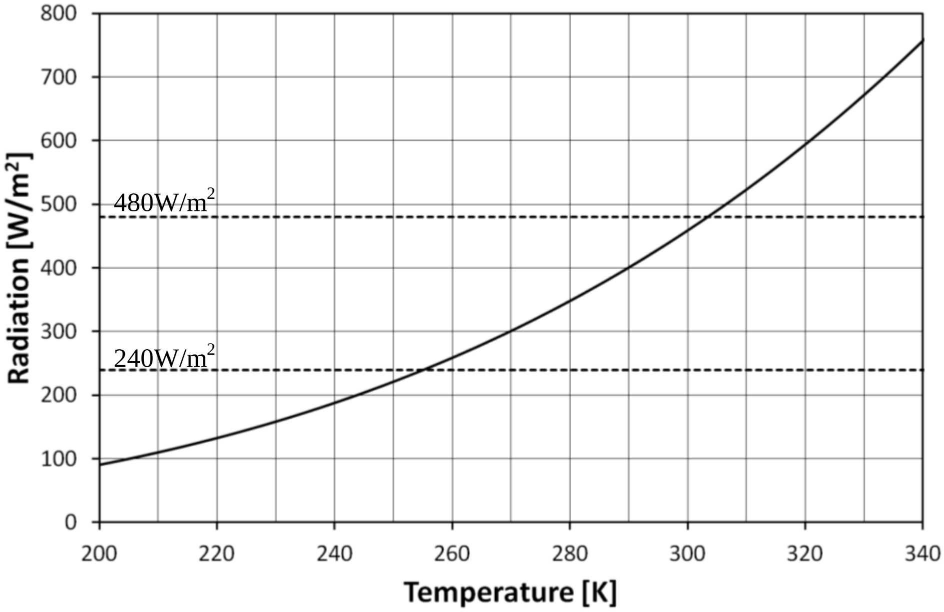


Figure 3: Stefan-Boltzmann's law

The terrestrial radiation follows this law.

3. Heat budget without greenhouse effect

The surface of a planet reaches an equilibrium state at a temperature where the terrestrial radiation F is equal to the net solar radiation S. Thus,

$$S = F$$

Substituting (1) to S, and (2) to F, the heat balance gives

$$\frac{(1-\alpha)S_0}{4} = \sigma T^4 \qquad \text{(A)}$$

Dividing both sides by σ, we obtain

$$\frac{(1-\alpha)S_0}{4\sigma} = T^4$$

By calculating the fourth root of both sides, we have

$$T = \sqrt[4]{\frac{(1-\alpha)S_0}{4\sigma}}$$

where T is defined as **effective radiation temperature**. When $S_0 = 1.37 \times 10^3 \mathrm{W/m^2}$ and $\alpha = 0.30$,

$$T = \sqrt[4]{\frac{(1-0.30)\times 1.37\times 10^{3}}{4\times 5.67\times 10^{-8}}} \cong 255[\mathrm{K}]\ \ (\cong -18[℃])$$

This value is much lower than the global mean surface temperature of the earth.

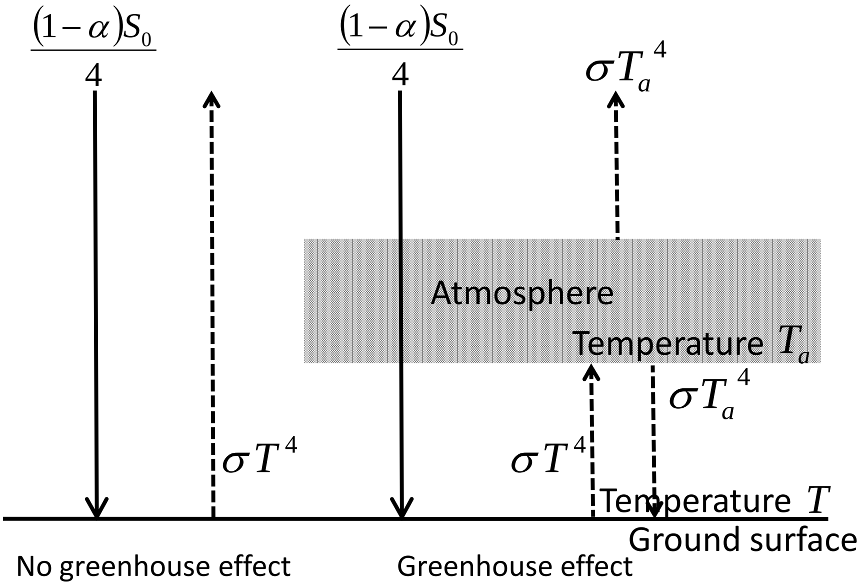


Figure 4: Schematic diagram of greenhouse effect

Table 1: Effective radiation temperature and mean surface temperature of the planets

	Mean distance from the sun (AU)	Solar radiation (W/m²)	Albedo	Effective radiation temperature (℃)	Mean surface temperature (℃)	Surface pressure (Earth = 1)	Main components
Mercury	0. 39	9100	0. 11	162	170		
Venus	0. 72	2600	0. 78	−49	460	90	CO_2
Earth	1. 00	1370	0. 30	−18	15	1	N_2、O_2
Mars	1. 52	580	0. 16	−58	−40	0. 006	CO_2
Jupiter	5. 20	50	0. 73	−185	−140		H_2、He

４． Greenhouse effect

4.1 Greenhouse gas

The earth has an atmosphere. The earth’s atmosphere is almost transparent for visible light. Thus, the solar radiation is not absorbed by the atmosphere. However, some components of the atmosphere such like water vapor (H_2O) and carbon dioxide (CO_2) are opaque for infrared radiation. Therefore, the terrestrial radiation is absorbed by the atmosphere. These components are called **greenhouse gas**.

4.2 Heat budget

Let us consider the heat budget under the existence of an atmosphere including greenhouse gas. Assume that the atmosphere is transparent for solar radiation, but is opaque for terrestrial radiation (see Fig. 4). First, the energy balance at the ground gives

$$\frac{(1-\alpha)S_0}{4} + \sigma T_a^{\ 4} = \sigma T^4 \qquad \text{(B)}$$

where T_a is the atmospheric temperature, and T is the temperature of the ground surface (see Fig. 4). In addition, the energy balance of the atmosphere is written

$$\sigma T^4 = 2\sigma T_a^{\ 4} \qquad \text{(C)}$$

By doubling (B), and adding it to (C), we can eliminate T_a and obtain

$$\frac{(1-\alpha)S_0}{2} = \sigma T^4$$

or

$$T = \sqrt[4]{\frac{(1-\alpha)S_0}{2\sigma}}$$

Then, (C) gives

$$T_a = \sqrt[4]{\frac{(1-\alpha)S_0}{4\sigma}}$$

For $S_0 = 1.37\times10^3 \text{W/m}^2$ and $\alpha = 0.30$,

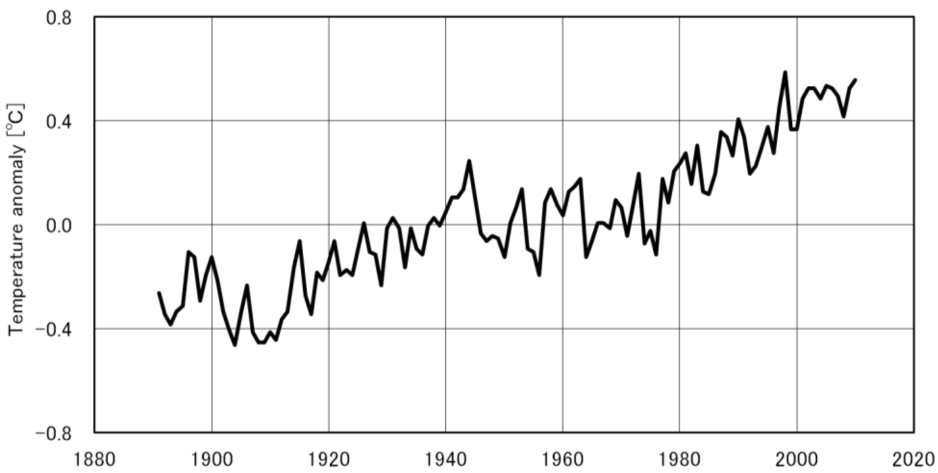
$$T = \sqrt[4]{\frac{(1-0.30)\times1.37\times10^3}{2\times5.67\times10^{-8}}} \cong 303[\text{K}] \ \ (\cong 30[℃])$$

$$T_a = \sqrt[4]{\frac{(1-0.30)\times1.37\times10^3}{4\times5.67\times10^{-8}}} \cong 255[\text{K}] \ \ (\cong -18[℃])$$

The value of surface temperature T ($\cong 30$[℃]) is much higher than that in the case of no greenhouse effect ($\cong -18$[℃]). Note that T_a is the air temperature averaged over the whole atmosphere (not the surface air temperature). In the real atmosphere, the air and the ground can exchange heat directly because they are in contact with each other. Taking this into consideration, we consistently understand that the global mean surface air temperature is 288K (≒15℃).

4.3 Greenhouse effect and global warming

In the previous subsection, greenhouse gas prevents the terrestrial radiation from escaping to space. Conversely, infrared radiation emitted from the atmosphere to the ground surface warms the surface. This is referred to as **greenhouse effect**. If the greenhouse gas is anthropogenically increased, the greenhouse effect will become stronger. The surface temperature will then rise. We call this anthropogenic change in climate the **global warming**.



(Japan Meteorological Agency)

Figure 5: Global mean surface air temperature